

Regularity for the Vlasov Equations in a Half Space

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ABSTRACT. Singularities will generally be created at the boundary for the coupled Vlasov-Poisson or Vlasov-Maxwell systems. However, global classical solutions are constructed for the Vlasov-Poisson system in a half-space, if either the particles have the same sign of charge or a neutral plasma reflects specularly at the boundary.

0. Introduction. Boundaries occur naturally in such physical problems as electric guns, particle accelerators and tokamaks. The particles have the same sign of charge in the first two examples while they are ideally neutral in the third example. The particles satisfy the Vlasov-Maxwell system inside the region. A simpler model is the Vlasov-Poisson system, where the speed of light is regarded as infinity. We consider the half space $[0, T] \times R_+^3 \times R^3$, where the boundary is $\{x_1 = 0\}$. The Vlasov-Poisson system takes the form:

$$\begin{aligned} \partial_t f^\beta + \frac{v}{m_\beta} \cdot \nabla_x f^\beta + e_\beta \nabla \varphi \cdot \nabla_v f^\beta &= 0, \\ f^\beta|_{t=0} &= f_0^\beta, \\ \Delta \varphi = \rho &= 4\pi \sum_\beta \int e_\beta f^\beta dx dv, \end{aligned} \tag{0.1}$$

where $1 \leq \beta \leq N$ is the index of the species of the particles. And, $f^\beta(t, x, v)$ is the distribution function of the density of the particles of species β , located at the point $x \in R_+^3$ with the velocity v . The constants e_β, m_β are the corresponding charge and mass of the species β , and $\nabla \varphi(t, x)$ is the self-consistent electric field. We shall combine this system with various boundary conditions. Particles can either be absorbed or specularly reflect at the boundary. Even in the simple one-dimensional Vlasov-Maxwell equations with boundary data, singularities generally occur, unless the electric field at the boundary has the correct sign. See [G2] for details.