

Continuity and Harnack's Inequality for Solutions of Elliptic Partial Differential Equations of Second Order

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ABSTRACT. We show local boundedness, continuity, Harnack's inequality and the property $|\nabla u|^2 \in K_n^{\text{loc}}(\Omega)$ for weak solutions u of $-\text{div}(A(x)\nabla u) + \mathbf{b}(x) \cdot \nabla u + V(x)u = 0$, where V and $|\mathbf{b}|^2$ belong to the local Kato class.

1. Introduction and main results. We consider the following elliptic equations of second order with real coefficients:

$$(1) \quad Lu \equiv -\text{div}(A(x)\nabla u(x)) + \mathbf{b}(x) \cdot \nabla u(x) + V(x)u(x) = 0 \quad \text{in } \Omega,$$

where Ω is a bounded domain in \mathbf{R}^n , $n \geq 3$ and $A(x) = (a_{ij}(x))$ satisfies

$$(2) \quad a_{ij}(x) = a_{ji}(x), \quad \lambda|\xi|^2 \leq \sum_{i,j=1}^n a_{ij}(x)\xi_i\xi_j \leq \lambda^{-1}|\xi|^2,$$

for some $\lambda \in (0, 1]$ and for every $x \in \Omega$, $\xi \in \mathbf{R}^n$. Define

$$\eta(f; r) = \sup_{x \in \mathbf{R}^n} \int_{B_r(x)} \frac{|f(y)|}{|x-y|^{n-2}} dy$$

and $\eta(f; r; G) = \eta(f\chi_G; r)$. Here χ_G is the characteristic function of G and $B_r(x) = \{y \in \mathbf{R}^n; |x-y| < r\}$ for $r > 0$. We say $V \in K_n(\Omega)$ if $\lim_{r \rightarrow 0} \eta(V; r; \Omega) = 0$ and say $V \in K_n^{\text{loc}}(\Omega)$ if $\lim_{r \rightarrow 0} \eta(V; r; \Omega_1) = 0$ for each bounded domain Ω_1 with $\bar{\Omega}_1 \subset \Omega$. The main assumptions of this paper are as follows:

Assumption A. $V, |\mathbf{b}|^2 \in K_n^{\text{loc}}(\Omega)$.