

Weyl Quantization and Tensor Products of Fock and Bergman Spaces

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ABSTRACT. Let V be a complex (flat) n -dimensional vector space and let $H_n = V \times \mathbb{R}$ be the Heisenberg group. We consider the tensor product of the Fock space on V by its complex conjugate. We present our observation that the polar decomposition of the diagonal operator (as an intertwining operator for the Heisenberg group actions) from the tensor product to the L^2 -space on V gives the Weyl transform. We then use this observation to study multiplicity-free actions of a compact group on V and the associated bounded spherical functions on the Heisenberg group. We give an easy proof of the Rodrigues type formula for the spherical functions previously obtained by Benson, Jenkins and Ratcliff. We generalize this consideration to the study of tensor products of weighted Bergman spaces on a bounded symmetric domain and thus generalize Weyl quantization to a curved setting. We study the Laguerre type functions on the domain and their spherical transforms, which in the rank one case are the continuous dual Hahn polynomials. By taking an appropriate zero-curvature limit we finally recover the flat case from the bounded case.

0. INTRODUCTION

Let V be a complex n -dimensional vector space equipped with the Lebesgue measure dz . The so-called Fock space \mathcal{F} on V is the Hilbert space of holomorphic functions on V that are square integrable with respect to the Gaussian measure on V . The Heisenberg group $H_n = V \times \mathbb{R}$ has a unitary representation $\pi =$