

On Uniform Approximation Problems and T -Invariant Algebras

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ABSTRACT. Let K be a compact subset of the complex plane \mathbb{C} and let $P(K)$ denote the uniform closure of analytic polynomials in z . Let A and B be two T -invariant algebras and let g be a C^1 function with compact support. In this paper, we prove that, for a large class of C^1 functions g , the inclusion $A \subset B \subset \text{clos}(P(K)g + A)$ implies $A = B$.

1. Introduction. Let Λ be a constant coefficient elliptic differential operator in R^n . For a compact subset $K \subset R^n$ let $H(K, \Lambda)$ and $h(K, \Lambda)$ denote the uniform closures in $C(K)$ of the set

$$\{f|_K : \Lambda f = 0 \text{ in some neighbourhood of } K\}$$

and the set

$$C(K) \cap \{\Lambda f = 0 \text{ in the interior of } K\},$$

respectively. One of uniform approximation problems is to characterize those K for which $H(K, \Lambda) = h(K, \Lambda)$.

A complete solution for $\Lambda = \Delta$ was obtained by Deny [D] and Keldysh [K] using a duality argument relying on classical potential theory. Let Cap denote the Wiener capacity in potential theory. Their theorem is as follows.

Theorem (Deny-Keldysh). *The identity $H(K, \Delta) = h(K, \Delta)$ occurs if and only if for each open ball B one has $\text{Cap}(B \setminus \text{int } K) = \text{Cap}(B \setminus K)$.*

Using a constructive scheme for uniform approximation (based on a localization operator), Vitushkin [Vi] solved in the sixties the problem for $\Lambda = \bar{\partial}$ (the Cauchy-Riemann operator in the plane).

Theorem (Vitushkin). *The identity $H(K, \bar{\partial}) = h(K, \bar{\partial})$ occurs if and only if for each open disc O one has $\alpha(O \setminus \text{int } K) = \alpha(O \setminus K)$, where α is the continuous analytic capacity (see [Vi]).*