

Pairs of Planes Which Are Not Size-Minimizing

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ABSTRACT. In this paper we use a simple modification of a family of surfaces defined in [Lr1] to prove one half of a conjecture by Frank Morgan concerning size-minimizing pairs of planes. The modification uses only “scissors and glue” to cut, fit together, and add in a flat disk. Each curved piece of a modified surface has the same shape as a piece of a surface of [Lr1].

1. INTRODUCTION

Area (also called mass) and size are two ways of measuring a surface. The difference is in how they count “multiplicities” on the surface. For example, if we construct a surface in \mathbb{R}^2 as a union of two overlapping disks, we may wish to think of the overlapping part as having multiplicity 2. Then the area, which counts multiplicity, will be $2\pi r^2$, while the size will be less, because it only counts the overlapping part once.

A surface is area-minimizing if it has least area among surfaces satisfying some constraint such as a fixed boundary. A surface with infinite area is called area-minimizing if every compact portion of the surface has least area given its boundary. Size minimization is defined similarly. Some area-minimizing surfaces are not size-minimizing, and vice-versa.

In 1981 Frank Morgan conjectured which pairs of oriented, m -dimensional planes through the origin in \mathbb{R}^n are area-minimizing. This conjecture was proved by complementary results of Dana Mackenzie [Mc] and the author [Lr1].

In a more recent paper, “Size-minimizing rectifiable currents,” Morgan proved a number of fundamental results about existence and regularity of size-minimizing surfaces, and classified the complete size-minimizing surfaces in \mathbb{R}^3 . He included a conjecture [M, 2.10] concerning which pairs of m -dimensional planes in \mathbb{R}^n are *size*-minimizing and which are not.

The criterion for minimizing area and the conjecture for minimizing size are both stated in terms of the “characterizing angles” between a pair of m -dimensional planes. Basic definitions can be found in a 1929 text by Sommerville