

# *Uniqueness of Renormalized Solutions in a $T$ -set for the $L^1$ -data Problem and the Link between Various Formulations*

JEAN MICHEL RAKOTOSON

ABSTRACT. We prove the uniqueness of the so-called renormalized solution in an appropriate  $T$ -set for the problem  $Au + a(u) = \mu \in L^1(\Omega)$ , where  $A$  is an operator satisfying Leray-Lions type conditions.

## INTRODUCTION

This paper deals essentially with the uniqueness of the solution  $u$  of the equation:

$$(1) \quad \begin{cases} -\operatorname{div}(\hat{a}(x, u, Du)) + \mathbf{a}(u) = \mu & \text{in } L^1(\Omega), \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Many authors solve this equation under various hypotheses on  $\hat{a}$  and  $\mathbf{a}$ , and mainly they consider a solution satisfying (1) in the sense of distribution. For such results see, for instance, [BS], [BG], [Ra1], [Ra2],... and the references therein.

Unfortunately, as pointed out by Serrin [S], the solution of such an equation is (in general) not unique. For this, he constructed a function  $u$  not identically zero and coefficients  $a_{ij}$  being bounded and not continuous, satisfying the following linear equation:

$$\operatorname{div}(A(x)Du) = 0$$

with  $A$  a matrix whose components are  $a_{ij}$ . The formulation is taken in the sense of distribution. The function  $u$  is in  $W_0^{1,1}(\Omega)$  and not in  $W_0^{1,2}(\Omega)$ . Later, Brézis and Strauss showed that the semi-linear equation:

$$\begin{cases} Lu + \mathbf{a}(u) = \mu & \text{in } L^1(\Omega), \\ u & \text{in } W_0^{1,1}(\Omega), \end{cases}$$