

On Fast Diffusion Nonlinear Heat Equations and a Related Singular Elliptic Problem

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ABSTRACT. We provide necessary conditions on u_0 for the solvability of the problem

$$\frac{\partial u}{\partial t} = \operatorname{div}(u^{m-1}\nabla u) \quad \text{in } \mathbf{R}^N \times (0, T),$$

$$u(x, 0) = u_0(x) \quad \text{in } \mathbf{R}^N,$$

where $T > 0$, $m < \min\{0, (N-2)/N\}$ or $m = 0$, $N = 2$. We also find both, necessary and sufficient conditions for the solvability of the elliptic problem

$$\Delta u + u^{-\nu} = f(x), \quad x \in \mathbf{R}^N,$$

where $\nu > 0$, $N \geq 2$, which is obtained by discretization of the parabolic problem when $\nu = -1/m$.

Introduction. Our aim in the first part of this paper is to provide a sharp necessary condition for the solvability of the Cauchy problem

$$(1.1) \quad \frac{\partial u}{\partial t} = \operatorname{div}(u^{m-1}\nabla u) \quad \text{in } \mathbf{R}^N \times (0, T),$$

$$(1.2) \quad u(x, 0) = u_0(x) \quad \text{in } \mathbf{R}^N,$$

where $T > 0$, $m \leq 0$ are given constants and u_0 is a nonnegative, locally integrable function. Equation (1.1) can also be written as

$$(1.3) \quad \frac{\partial u}{\partial t} = \Delta \varphi_m(u) \quad \text{in } Q_T,$$