

# *A Structure Theorem for Singly Generated Dual Uniform Algebras*

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**1. Introduction.** This paper deals with the class of dual Banach algebras. These are Banach algebras that are duals of Banach spaces and for which the multiplication is separately weak\* continuous.

In [8], G. Cassier proved that if the dual algebra generated by a Hilbert space operator  $T$  is uniform, then  $T$  has a nontrivial invariant subspace. This result was obtained as a corollary of a decomposition theorem for this class of dual algebras. This theorem is proved in [8] under some extra conditions that are dropped in [9].

Our aim is to give a similar decomposition in the abstract case of dual Banach algebras. The main result of the paper is the following (the notations will be reviewed below).

**Theorem 1.1.** *Let  $\mathcal{A}$  be a singly generated dual uniform algebra with separable predual  $X$ . Let  $a \in \mathcal{A}$  be a weak\* generator for  $\mathcal{A}$  and let  $K$  denote the spectrum of  $a$  in  $\mathcal{A}$ . Then:*

- (a)  *$R(K)$  is Dirichlet*
- (b) *there exists a positive measure  $\mu$  on  $\partial K$ , singular with respect to harmonic measure on  $\partial K$  such that the natural map  $\text{Rat}(K) \ni f \rightarrow f(a) \in \mathcal{A}$  extends to an isometric isomorphism and weak\* homeomorphism  $\Phi : L^\infty(\mu) \oplus H^\infty(\text{Int } K) \rightarrow \mathcal{A}$ .*

The proof of this theorem will be given in Section 4. The study of (non-selfadjoint) dual operator algebras was initiated by S. Brown in [3] in connection with the invariant subspaces problem. The dual algebras techniques were then applied to various classes of operators, leading to several important results concerning the invariant subspaces and reflexivity of operators. See [2] for the state of the theory until 1985 and also [1], [4], [5], and [10] for more recent results.