

On Nonpositive Curvature Functions on Noncompact Surfaces of Finite Topological Type

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ABSTRACT. We study the conformal deformation equation $\Delta u - 2k_g + 2Ke^u = 0$ on noncompact surfaces of finite topological type with K nonpositive. Sharp existence and nonexistence results are obtained. There are two kinds of nonexistence results for this equation with K nonpositive. The first is of a global geometric nature and therefore it should be determined by the topology of the surface. We show that it is the Euler characteristic of the surface that determines this kind of nonexistence result for the class of noncompact conformal surfaces of finite topological type with no hyperbolic ends. This also reveals the topological significance of the results previously obtained by D. H. Sattinger and W. M. Ni on the Euclidean plane \mathbb{R}^2 . Its most general form is stated in Theorem 0.9. The second kind of nonexistence result is a purely geometric property of the ends. Typical examples are theorems 0.5 and 0.6. Thus if a conformal surface contains a hyperbolic end, topology plays no role in the study of the equation. In other words, if K is a nonpositive function on M , then the above equation has a solution on M if and only if restricted to each end, this equation has a solution.

Introduction. Given a conformal structure \mathcal{C} on a 2-dimensional surface M , a basic problem in Riemannian geometry is to describe the set of conformal curvature functions $K_{\mathcal{C}} = \{K \mid K \text{ is the curvature function of a Riemannian metric representing the conformal structure } \mathcal{C}\}$. Fix a metric $g \in \mathcal{C}$. Then this problem is equivalent to the solvability of the following nonlinear elliptic equation, that is, a function K on M is in $K_{\mathcal{C}}$ if and only if the elliptic equation

$$(*) \quad \Delta u - 2k_g + 2Ke^u = 0$$