

# *Singularities and Self Intersections of Curves Evolving on Surfaces*

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ABSTRACT. It is proven that whenever a closed curve evolving by an arbitrary uniformly parabolic equation  $\gamma_t = V(T, k)N$  on a Riemannian manifold  $M$  develops a singularity, it either shrinks to a point or loses a self intersection.

To aid in the calculations, it is first shown that locally it is enough to consider the  $\mathbb{R}^2$  case. Given a local conformal map from a neighborhood of  $M$  into another manifold  $M'$  (which we later set to  $\mathbb{R}^2$ ), the image on  $M'$  of  $\gamma$  also obeys an arbitrary uniformly parabolic equation.

**1. Introduction.** In a sequence of two papers ([A1], [A2]) Sigurd Angenent proved basic results for the evolution on a Riemannian manifold  $M$  of a curve  $\gamma$  whose velocity vector at each point can be written as

$$(1-1) \quad \frac{\partial}{\partial t} \gamma = V(T, k)N.$$

$T$  and  $N$  denote the unit tangent and normal vectors to  $\gamma$ , and  $k$  the curvature of  $\gamma$ . (1-1) is general enough to unify a handful of specific evolutions which have recently received attention, the most well known of these being the curve shortening flow

$$(1-2) \quad \frac{\partial}{\partial t} \gamma = kN.$$

See [A1] for mention of other evolutions. A more recent example which fits into this category is the flow by curvature in relative geometries studied by Gage [G2]. We prove in this paper that if  $V$  satisfies certain conditions, then whenever a curve  $\gamma$  evolving by (1-1) develops a singularity it loses at least one self-intersection.