

On a Region Free from the Poles of the Resolvent and Decay Rate of the Local Energy for the Elastic Wave Equation

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ABSTRACT. We deal with the elastic wave equation with the Neumann boundary condition in an exterior domain. In this equation, it is well-known that there exist the Rayleigh surface waves propagating along the boundary. Hence, we can expect that any pole of the resolvent, except that corresponding to the Rayleigh surface waves, does not appear near the real axis when all waves except the Rayleigh surface waves go away from the boundary. We justify this expectation and also obtain an estimate of the resolvent for some class of anisotropic elasticities. As an application of these results, in the isotropic case, we get information about decay-speed of the local energy of the solution for the time dependent problem.

0. Introduction. Let Ω be an exterior domain in \mathbf{R}^n ($n \geq 3$) with C^∞ and compact boundary Γ . We define differential operators $A^0(\partial_x)$ and $A^1(x, \partial_x)$ as

$$A^0(\partial_x)u = \sum_{i,j=1}^n \partial_{x_i}(a_{ij}^0 \partial_{x_j} u), \quad A^1(x, \partial_x)u = \sum_{i,j=1}^n \partial_{x_i}(a_{ij}^1(x) \partial_{x_j} u),$$

where $u = {}^t(u_1, u_2, \dots, u_n)$ and $a_{ij}^0, a_{ij}^1(x)$ are $n \times n$ -matrices. The (p, q) -component a_{ipjq}^0 of a_{ij}^0 is a real constant and the (p, q) -component $a_{ipjq}^1(x)$ of $a_{ij}^1(x)$ is a real-valued C^∞ function on \mathbf{R}^n whose support is compact. We set $a_{ij}(x) = a_{ij}^0 + a_{ij}^1(x)$ and

$$A(x, \partial_x)u = A^0(\partial_x)u + A^1(x, \partial_x)u.$$

Thus each component $a_{ipjq}(x)$ of $a_{ij}(x)$ is of the form $a_{ipjq}(x) = a_{ipjq}^0 + a_{ipjq}^1(x)$.