

On the Trace Inequalities for Hardy-Sobolev Functions in the Unit Ball of \mathbf{C}^n

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ABSTRACT. We characterize “invariant measures” on the spheres in \mathbf{C}^n for which the trace inequality,

$$\int_{\mathbf{S}} (M_\alpha F)^p d\mu \leq C \|F\|_{H_\beta^p}^p,$$

holds for holomorphic functions F in the Hardy-Sobolev spaces H_β^p , where $1 < p < \infty$, and M_α is the admissible maximal function. In contrast to the known imbedding theorems for Euclidean Sobolev spaces, we obtain characterizations that distinguish the cases $1 < p \leq 2$ and $2 < p < \infty$. Applications to exceptional sets are also given.

0. Let \mathbf{S} be the boundary of \mathbf{B}_n , the unit ball in \mathbf{C}^n , and $d\sigma$ the rotation invariant measure defined on \mathbf{S} . We define the non-isotropic potential of a function $f \in L^1(d\sigma)$ by

$$I_\beta f(\eta) = \int_{\mathbf{S}} \frac{f(\zeta)}{|1 - \langle \eta, \zeta \rangle|^{n-\beta}} d\sigma,$$

where $\eta \in \mathbf{S}$ and $0 < \beta < n$.

If $1 < p < \infty$, let L_β^p be the space of potentials $F = I_\beta f$ where $f \in L^p(d\sigma)$ with norm

$$\|F\|_{L_\beta^p} = \|f\|_{L^p(d\sigma)}.$$

In the case when β is an integer, L_β^p coincides with a (non-isotropic) Sobolev space on \mathbf{S} . We will also need the Hardy-Sobolev spaces, H_β^p ($0 < \beta, p < \infty$), of functions F holomorphic in the unit ball. Let

$$F(z) = \sum_k f_k(z)$$