

On the Neumann Problems for Complex Monge-Ampère Equations

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ABSTRACT. The existence and uniqueness of classical plurisubharmonic solution for the Neumann problem of complex Monge-Ampère equations in certain strictly pseudoconvex domain in \mathbb{C}^n are proved.

1. Introduction. Let Ω be a bounded strictly pseudoconvex domain in \mathbb{C}^n with C^2 boundary. Let f be a non-negative function on $\Omega \times \mathbb{R} \times \mathbb{R}^{2n}$. Let $\varphi(z, u)$ be function on $\partial\Omega \times \mathbb{R}$. We shall study the existence, uniqueness and regularity of plurisubharmonic solutions of the complex Monge-Ampère equation:

$$(1.1) \quad \det \left(\frac{\partial^2 u}{\partial z_i \partial \bar{z}_j} \right) = f(z, u, \nabla u), \quad z \in \Omega,$$

associated to the Neumann boundary condition:

$$(1.2) \quad D_\nu u = \varphi(z, u), \quad \text{on } \partial\Omega,$$

where D_ν denotes the normal derivative along $\partial\Omega$.

The Dirichlet problem of the complex Monge-Ampère equation (1.1) with Dirichlet boundary condition $u = \varphi$ has received considerable study. In 1976, Bedford and Taylor [BT1] considered weak plurisubharmonic solution of complex Monge-Ampère equations: $\det(\partial^2 u / \partial z_i \partial \bar{z}_j) dV = \mu$, where μ is a bounded non-negative Borel measure. Also in [BT2]–[BT3], they studied the Dirichlet problem by using the Perron-Bremermann family method, thereby proving the existence and uniqueness of weak solutions. They have successfully proved a global Lipschitz regularity for plurisubharmonic solution when Ω is a bounded strictly pseudoconvex domain in \mathbb{C}^n . S. Cheng and S. Yau [CY] solved the Dirichlet problem represented by equations (1.1) with $f = e^u$, $u = \infty$ on $\partial\Omega$, obtaining a solution $u \in C^\infty(\Omega)$. In the non-degenerate case ($f > 0$), the existence and uniqueness of a classical plurisubharmonic solution of the Dirichlet