

L^p -Integrability of the Second Order Derivatives for the Neumann Problem in Convex Domains

VILHELM ADOLFSSON & DAVID JERISON

ABSTRACT. We give estimates in L^p , $1 < p \leq 2$, for the second order derivatives of the solution to the inhomogeneous Neumann problem in a convex domain. We also show that there is an atomic decomposition of functions in the Hardy space H^1 that vanish outside a domain if and only if the domain is a uniform domain in the sense of P. W. Jones.

Introduction. The purpose of this paper is to prove that for a convex domain Ω in \mathbf{R}^n , the second order derivatives $\nabla^2 u$ of the solution u to the Neumann problem for the Poisson equation with data f , are in $L^p(\Omega)$ if $f \in L^p(\Omega)$ with $1 < p \leq 2$. To avoid technicalities we assume throughout the paper that $n \geq 3$. The main result of the paper is:

Theorem. *Suppose Ω is a bounded convex domain in \mathbf{R}^n and $f \in L^p(\Omega)$, $1 < p \leq 2$ and $\int_{\Omega} f = 0$. Then there exists a solution u in $W^{2,p}(\Omega)$ solving*

$$\begin{cases} \Delta u = f & \text{in } \Omega, \\ u_{\nu} = 0 & \text{on } \partial\Omega, \end{cases}$$

and $u_{\nu} = \nu \cdot \nabla u$ denotes the (outer) normal derivative of u . Moreover,

$$\int_D |\nabla^2 u|^p dx \leq C \int_D |f|^p dx,$$

where ∇^2 denotes the second order derivatives and the constant C depends only on the Lipschitz constant of the domain.