

Averaged Motion of Charged Particles under Their Self-Induced Electric Field

AVNER FRIEDMAN & CHAOCHENG HUANG

ABSTRACT. In this paper we consider the averaged equations for a large number of small balls of uniform mass and charge moving under the force of their self-electric field. These equations are

$$\Delta\varphi(x,t) = -P(x,t), \quad d^2\psi/dt^2 = -\nabla\varphi(\psi,t)$$

subject to $\psi(x,0) = x$, $\psi_t(x,0) = \psi_1(x)$ where φ is the electric potential and P is the limit concentration of the small balls as their number increases to infinity (and their radius goes to zero). The evolution of P is given by

$$P(x,t) = P_0(\psi^{-1}(x,t))J(\psi^{-1})(x,t)$$

where P_0 is the initial concentration, ψ^{-1} is the inverse of the mapping $x \rightarrow \psi(x,t)$ and $J(\psi^{-1})$ is its Jacobian. It is proved that if the initial data are in $C^{1+\alpha}$ then there exists a unique local solution with ψ in $C^{1+\alpha}$. The solution can be extended globally in time as long as ψ and ψ^{-1} remain uniformly bounded in C^1 . There are however smooth initial data for which a global solution does not exist. One of the main results of the paper is that if $|P_0 - \chi_{B_1}|$ and $|\psi_1(x) - b_0x|$ are small enough, where χ_{B_1} is the characteristic function of the unit ball and b_0 is a positive real number, then there exists a unique global solution.

1. The averaged equations. Let Ω be a domain in \mathbb{R}^3 and consider disjoint balls $B_\varepsilon(x_i^\varepsilon(t))$ in Ω of radius ε and centers $x_i^\varepsilon(t)$ which vary in time t . We assume that each ball carries electric charge that is uniformly distributed within the ball, with total charge equal to its volume times a fixed constant. We denote by $\varphi_\varepsilon(x,t)$ the electric potential in Ω ; it is generated by the electric