

Uniqueness of Positive Solutions for Nonlinear Cooperative Systems with the p -Laplacian

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ABSTRACT. In this article we obtain the existence and uniqueness of a positive solution to the following strictly cooperative elliptic system,

$$\begin{cases} -\Delta_p u_i = \sum_{j=1}^n a_{ij} |u_j|^{p-2} u_j + f_i(x, u_1, \dots, u_n) & \text{for } x \in \Omega; \\ u_i = 0 & \text{on } \partial\Omega; i = 1, \dots, n. \end{cases}$$

Here, Ω is a bounded domain in \mathbb{R}^N whose boundary $\partial\Omega$ is of class $C^{2,\alpha}$ for some $\alpha \in (0, 1)$, Δ_p denotes the p -Laplacian defined by $\Delta_p u \stackrel{\text{def}}{=} \operatorname{div}(|\nabla u|^{p-2} \nabla u)$ for $p \in (1, \infty)$, and the coefficients a_{ij} ($1 \leq i, j \leq n$) are assumed to be constants satisfying $a_{ij} > 0$ for $i \neq j$ (a strictly cooperative system) and $a_{ii} < 0$. We assume that the functions $f_i(x, u_1, \dots, u_n)$ satisfy $f_i \in C^\alpha(\overline{\Omega} \times \mathbb{R}_+^n)$, $\partial f_i / \partial u_j \in C(\Omega \times (0, \infty)^n)$ and $f_i \geq 0$, $\partial f_i / \partial u_j \geq 0$ ($1 \leq i, j \leq n$), and each $\lambda \mapsto \lambda^{1-p} f_i(x, \lambda u_1, \dots, \lambda u_n)$ is a strictly monotone decreasing function of $\lambda \in (0, \infty)$. Our methods combine weak and strong comparison principles with standard tools for strongly subhomogeneous monotone mappings.

1. Introduction. In this article we study the following strictly cooperative elliptic system,

$$(1.1) \quad \begin{cases} -\Delta_p u_i = \sum_{j=1}^n a_{ij} \psi_p(u_j) + f_i(x, u_1, \dots, u_n) & \text{for } x \in \Omega; \\ u_i = 0 & \text{on } \partial\Omega; i = 1, \dots, n. \end{cases}$$