

Peak Functions on Weakly Pseudoconvex Domains

Jiye Yu

ABSTRACT. The main result of this paper gives a new sufficient condition for a boundary point of a domain to be a peak point for the algebra of holomorphic functions that are continuous on the closure of the domain. The condition is that the boundary is pseudoconvex there, and the Catlin's multi-type agrees with D'Angelo's q -types. We also provide other equivalent formulations.

Introduction. Let $\Omega \subset \mathbb{C}^n$ be a domain with a boundary point p . We say that p is a (continuous) *peak point* for Ω if there is a holomorphic function f on Ω which is continuous on the closure $\bar{\Omega}$ such that: $f(p) = 1$ and $|f(z)| < 1$ on $\bar{\Omega} \setminus \{p\}$. The function f is called a *peaking function*. The existence of peaking functions has many important applications in the function theory of several complex variables, we refer the reader to [BF1-2], [BP1-2], [CH], [F], [KR2], [RA], [YU4], to name only a few.

The basic problem here is to determine sufficient conditions for a boundary point to be a peak point. The maximum principle implies that if $p \in b\Omega$ is a peak point, then the boundary of Ω cannot contain any nontrivial complex variety through p . Thus some sort of finite type condition is necessary. There are several well-known finite type conditions available: Kohn's type [KO1], [B-G], [BL2], D'Angelo's 1-type [DA1] and Catlin's multitype [CA1]. By finite type we mean that the D'Angelo's 1-type is finite.

When the domain Ω is strongly pseudoconvex, the problem has long been well understood [HS1], [PF], [GR]. In case the domain is weakly pseudoconvex of finite type, very little is known. Significant progress has been made in two dimensions and in domains whose boundary geometry is rather simple. If $\Omega \Subset \mathbb{C}^2$, Bedford and Fornæss [BF1] showed that any boundary point of finite type is a peak point. This result was reestablished using a different method by Fornæss and Sibony in [FS]. Recently, Noell [NO] has extended the Bedford-Fornæss technique to a class of model domains of finite type with homogeneous defining functions. On the other hand, McNeal and Fornæss [FM] found the