

# The Range of the Complex Monge-Ampère Operator

SŁAWOMIR KOŁODZIEJ

ABSTRACT. We discuss the Dirichlet problem for the complex, non-homogeneous Monge-Ampère equation, finding some sufficient conditions for the right hand side to be the Monge-Ampère mass of a bounded plurisubharmonic function.

**Introduction.** We study the range of the complex Monge-Ampère operator  $u \rightarrow (dd^c u)^n$  acting on bounded plurisubharmonic functions which are defined in a strictly pseudoconvex subset of  $\mathbb{C}^n$ . For such functions,  $(dd^c u)^n$  is by [BT1] a (well defined) Borel measure. This measure is always dominated by the relative capacity of Bedford and Taylor (see [BT2], [B]),

$$\text{cap}(E, \Omega) = \sup \left\{ \int_E (dd^c u)^n, u \in \text{PSH}(\Omega), -1 < u < 0 \right\}.$$

If  $-M < u < 0$ , then obviously

$$\int_E (dd^c u)^n \leq M^n \text{cap}(E, \Omega).$$

Thus, in particular,  $(dd^c u)^n$  is zero on pluripolar sets. However, the inequality

$$(0.1) \quad \mu(E) \leq C \text{cap}(E, \Omega)$$

is not sufficient to ensure the existence of  $u \in \text{PSH}(\Omega) \cap L^\infty(\Omega)$  such that

$$(0.2) \quad (dd^c u)^n = d\mu$$

(see Example 2 in the last section). One would like to find a characterization of the image of the set of bounded plurisubharmonic functions through the Monge-Ampère operator.

A softer problem, but also open, is the following: Whether the existence of a subsolution  $v \in \text{PSH}(\Omega) \cap L^\infty(\Omega)$  with  $(dd^c v)^n \geq d\mu$  implies the existence of  $u$  satisfying (0.2)? (see [B], [CK], [CP], [CS]).