

# *Carathéodory's Temperature Equations*

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**Introduction.** In his first paper [2] on axiomatization of thermodynamics, C. Carathéodory stated that if a system of equations

$$\begin{aligned} &F_{1,2}(x_1, x_2) = 0 \\ (1) \quad &F_{1,3}(x_1, x_3) = 0 \\ &F_{2,3}(x_2, x_3) = 0 \end{aligned}$$

have the property that any two of them imply the other, then there exist functions  $f_i$  such that the graph of the system (1)—*i.e.* the set of all points  $(x_1, x_2, x_3)$  whose coördinates satisfy it—is the same as the graph of the system

$$(2) \quad f_1(x_1) = f_2(x_2) = f_3(x_3).$$

In a later paper [3] he used a similar system which will be described soon. Various commentaries by physicists, [1], [4], [5], also use one or the other of these systems, but neither the original papers nor the commentaries contain a proof of this statement.

In these applications each  $x_i$  stands for a set of  $m_i$  real variables which describe the state of a thermodynamic system  $S_i$ ,—in the simplest cases, just the volume and pressure of a gas. The equations  $F_{i,j}(x_i, x_j) = 0$  are the conditions that the systems  $S_i$  and  $S_j$  be in equilibrium, and the condition on the system (1) corresponds to the physical axiom: *if two systems are both in equilibrium with a third, then they are in equilibrium with each other.* The  $f_i$  are required to be real valued differentiable functions and  $f_i(x_i)$  is, under a preliminary definition which is further restricted later, called the *temperature*.

Clearly systems (1) can occur in any situation—osmosis, for example—which