

# *Uniqueness of Axially Symmetric Flows with Free Boundaries*

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**Introduction.** Many of the notable successes in the hydrodynamics of plane flows rest on the power of function-theoretic methods that are inapplicable to three-dimensional flow problems. In particular, the free boundary problems of hydrodynamics, which have been successfully studied in the plane by methods of conformal mapping, have resisted solution in the case of axial symmetry. To date the only noteworthy theoretical results on the latter problem seem to be Levinson's formula for the asymptotic shape of the axisymmetric infinite cavity [1]<sup>1</sup> and Friedrichs' proof by a variational method of local uniqueness of the jet flow from a channel [2].

In papers marking important advances in the plane free boundary problem, Lavrentieff [3] and Serrin [4] reduced the previously formidable uniqueness problem to comparatively simple geometric considerations centering about some hydrodynamic comparison theorems. These theorems show the qualitative dependence of the flow speed, at a fixed boundary point of a flow region, on variations of the region preserving certain properties of the flow in the large. The simplicity of the comparison theorems and of the methods based on them suggest an approach to the axially symmetric free boundary problem. The purpose of this paper is to prove the analogue of two of these theorems for axially symmetric flows (Theorems 1 & 2), and by means of them to establish, under certain broad assumptions, uniqueness in the large for two of the classical axisymmetric free boundary problems, namely, the problem of the Helmholtz infinite cavity or wake (Theorem 3) and the problem of jet flow out of a channel or vessel (Theorem 4). While further applications suggest themselves, we re-

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<sup>1</sup> For additional results on the asymptotic shape of the cavity, see F. SCHEID, *Amer. Jour. Math.* **72**, 485-501 (1950), and M. GUREVICH, *Prikl. Mat. i. Mekh.* **11**, 97-104 (1947).