

Spinor Space and Line Geometry II

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Synopsis. In the first paper with the same title [1], which will be referred to as SI, we discussed the representation and the mapping of the ideal space L_3 of the centered vector space R_4 of the theory of special relativity onto the spinor space S_3 . It turned out that a congruence K as well as a linear complex Γ played a very important role. In the first part of this paper (§§1–5) we shall use these line configurations in S_3 as “absolute” configurations in S_3 , where now spinor lines (and not spinors) will be considered as elements. Thus a line possesses an invariant, a couple of lines possess a couple of invariants. These invariants lead to a “line metric” dealt with in the two last sections of the first part. In the second part (§§6, 7) we use the biaxial involution in S_3 dealt with in SI for a geometric interpretation of Dirac equations. Moreover, this biaxial involution in S_3 is a map of a reflection in L_3 and this reflection leads in a natural way to Schrödinger’s relativistic wave equation.

While in the present paper we deal with the spinor space S_3 from the point of view of its lines, in the next paper we will discuss the analysis of S_3 from the point of view of its spinors.

0. Generalities

Our considerations are based on results of SI. In particular we shall use its symbolism as well as its language. For the reader’s convenience we repeat that the congruence K is the intersection of the complexes

$$(0.1) \quad \Gamma^{ab}(1, 1, 0, 0, 0, 0), \quad * \Gamma^{ab}(-1, 1, 0, 0, 0, 0),$$

and that its axes are

$$(0.2) \quad {}^1\phi^{ab}(0, 1, 0, 0, 0, 0), \quad {}^2\phi^{ab}(1, 0, 0, 0, 0, 0).$$

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