

*The General Problem of the Transformation
of the Equations of Dynamics*

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INTRODUCTION

In this paper we present essentially the detailed demonstrations of the results announced by A. Lichnerowicz¹ concerning the problem of the transformation of dynamical systems with constraints dependent on time but where the expressions for the kinetic energies and the generalized forces do not depend explicitly on time. The last restriction must be imposed from the very form of the problem.

Briefly, the old problem of the transformation of the equations of dynamics can be stated as follows: *Given two dynamical systems D and E which can be referred to the same set of generalized coordinates, what are the necessary and sufficient conditions for all trajectories of E to be trajectories of D (independently of their time parameterizations)? In particular, what are the conditions for the trajectories of the two systems to be identical?* This problem was studied without complete success by Painlevé, Levi-Civita, Stäckel² and others. T. Y. Thomas³ has completely solved the problem in the case of dynamical systems with constraints independent of time.

Our approach has been similar in general to that of Thomas. Starting with the expressions for the kinetic energies of the two systems and associating with each

¹ C. R. Acad. Sc., **222**, 1946, p. 649.

² PAINLEVÉ, Journ. de Math., 4^e série, **10**, 1894, p. 42; LEVI-CIVITA, Ann. di Matem., 2^e série, **24**, 1896, p. 10.

³ Journ. of Math. and Phy., **25**, 1946, p. 191.