

Thin Liquid Jets

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Dedicated to the memory of P. F. NEMÉNYI

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1. Introduction. In 1877 Boussinesq [1] obtained equations of motion for thin liquid jets (water bells), taking into account surface tension and body forces. Using these equations, he was able to give a theoretical explanation of certain phenomena observed by Savart [2]. He considered only the case where the streamlines are meridians on a surface of revolution. His theory predicts that, in the absence of body forces, the streamlines will be catenaries. This paper also contains a stability investigation.

We shall generalize Boussinesq's equations to describe bells not axially symmetric. We derive a rather limited class of solutions for the case where the body force vector is constant and a much larger class for the case when the body force vanishes identically. An important subclass of these latter solutions can be obtained by a simple geometric construction. We derive a substitution principle, valid when the body force vanishes identically, this principle enabling one to construct an infinite number of solutions once one is known. A few facts concerning characteristic curves are established.

Consider the stationary motion of a liquid jet in a homogeneous medium, \mathcal{M} , which is at rest. In the jet let p denote pressure, ρ the density, f_i the components of the body force per unit mass, and u_i the velocity components. Assuming the liquid non-viscous and incompressible, the equations of motion are

$$(1.1) \quad p_{,i} + \rho u_{i,j} u^j = \rho f_i,$$

$$(1.2) \quad u^i_{,i} = 0,$$