

Two Hydrodynamic Comparison Theorems

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1. Recently it has become evident that in the study of the classical problem of flows with free boundaries, certain hydrodynamic comparison theorems can be employed to great advantage. The first statement and use of these results occurred in the work of Lavrentieff [1], and more recently they have been presented in simpler form by Serrin [2]. The extension of these theorems to axially symmetric flows was inaugurated by Gilbarg [3], when he proved the analogues of Lavrentieff's Theorems 1 and 1' for such flows.

In the present paper we obtain for axially symmetric flows the analogues of Theorem 2 of [2]. It is important to note that while functiontheoretic methods give the simplest approach to the plane problem, in axial symmetry an entirely different, and seemingly more general, method is necessary. Finally we prove two corollaries which give quantitative estimates of flow speeds at inflection points on streamlines. These corollaries were given by Lavrentieff (Theorems 3 and 3'), but his proofs are hard to understand, apparently for lack of theorems corresponding to those of the present paper. It is assumed that the reader is familiar with [3], for many of our arguments have been shortened by references to proofs there.

In a second paper the author intends to apply the present results to obtain a number of results concerning the geometric behavior of free streamlines, including a determination of the obstacle of given dimensions with least cavity or wake drag.

2. We consider first axially symmetric irrotational flows of an ideal incompressible fluid. These flows are described by a stream function $\Psi(x, y)$, which is

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