

Uniqueness Theorems for Certain Parabolic Problems

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Communicated by M. H. MARTIN

Two uniqueness theorems are presented for parabolic problems on a semi-infinite strip $S: 0 \leq x \leq 1, 0 \leq t$. The differential equation is $u_{xx} - u_t = N(x, t, u, u_x)$, where N may be nonlinear. The boundary conditions are linear in u_x , may contain u_t linearly, and may be nonlinear in u .

It will be proved first that certain problems of the form

$$u_{xx} - u_t = N(x, t, u, u_x), \quad 0 \leq x \leq 1, 0 < t,$$

$$u_{x0} - a_0(t)u_{x0} = n_0(t, u_0), \quad x = 0, 0 < t,$$

$$u_{x1} + a_1(t)u_{t1} = n_1(t, u_1), \quad x = 1, 0 < t,$$

$$u(x, 0+) = g(x),$$

(where subscript letters denote partial derivatives) cannot have two solutions in a certain natural class of functions u if the functions $a_0(t)$ and $a_1(t)$ are non-negative. Here $u_j \equiv u(j, t)$ for $j = 0, 1$, and similarly for u_{xj} and u_{tj} ; the subscript numbers on a and n are mere indices.

Let P denote any problem of the above form in which the following requirements are met:

1. $a_j(t)$ are continuous for $t > 0$;
 2. $n_j(t, w)$ are continuous and have partial derivatives n_{jw} that are continuous for $0 < t$ and $-\infty < w < \infty$, and are bounded uniformly in t ;
 3. $N(x, t, y, z)$, N_y , and N_z are continuous for $0 \leq x \leq 1, 0 < t$, and $-\infty < y, z < \infty$, and are bounded uniformly in x and t ;
 4. $g(x)$ is continuous.
- “Uniform boundedness” here means specifically that $|w| \leq \rho$ implies $|n_{jw}| \leq$