

*Variational Problems in the Theory of
Elliptic Partial Differential Equations**

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1. Introduction. In various problems of mathematical physics one is led to study functionals of closed curves or surfaces, such as, for example, the electrostatic capacity, the virtual mass, the eigenvalues of certain boundary value problems, etc. In order to obtain estimates for these functionals in terms of simpler geometric quantities or of other functionals, we study extremum problems for these expressions. It is now a remarkable fact that in almost all cases where an exact solution of an extremum problem can be given, the solution curve or surface has analytic character. In this paper we wish to develop through a few examples of general interest some mathematical tools by means of which the existence of analytic extremal curves for certain of these minimum problems involving linear elliptic partial differential equations with analytic coefficients and in two independent variables can be verified. Our main interest in the approach to such problems which we present here arises from its wide applicability to a varied field of classical questions in elliptic partial differential equations. We treat in detail only a few special variational problems for the eigenvalues of the vibrating membrane, but it is our hope that the reader will find in our concrete discussion of such examples the outline of the general principles which are the main center of attraction.

There are a few basic ideas involved in our variational and analytical investiga-

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