

*Comparison Methods in the Theory of Subsonic Flows**

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Introduction. We shall be concerned with several qualitative results in the theory of subsonic flows. Our methods will be based on comparison of stream functions for flows having a common subregion of definition and certain general properties in the large. These comparison methods have been singularly successful when applied to the free boundary problems of hydrodynamics (as in [1], [2], [3]) and recommend themselves to the problems of compressible fluids because the difficulties raised by nonlinearity can be circumvented by these means without recourse to the hodograph plane, and also because the simplicity and generality of the arguments are well suited to a theory so lacking in rigorously established general theorems.

In this paper we first prove several theorems on the monotonic dependence of the boundary flow speed on the free stream velocity in flow past a wall and past a symmetric body (Theorems 1, 2, 3). By means of a new theorem of Phragmén-Lindelöf type on the behavior at infinity of bounded solutions of elliptic partial differential equations (Theorem 4), we then prove the uniqueness with respect to free stream Mach number and maximum local Mach number of subsonic flows past a certain class of walls, including as special case the symmetric flow past a finite body (Theorems 6, 7). Theorem 8 extends to subsonic flows a comparison theorem first stated by Lavrentieff [1] for incompressible fluids. We see in Theorems 9 and 10 that all our results for flows past a wall carry over, more simply and more generally, to flows in a channel. Finally we consider flows with axial symmetry—that we can do so at all is one of the distinctive features of the comparison method—and show that the theorems on monotonic dependence and on flows in a channel remain true without essential modification.

As application of these theorems we obtain immediately the extension to subsonic flows of a number of the previous results on free boundaries in incompressible fluids. In a later work, in collaboration with M. Shiffman, we shall apply

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