## Axiomatic Foundations of Classical Particle Mechanics<sup>1</sup>

## J. C. C. McKINSEY, A. C. SUGAR, & PATRICK SUPPES

Stanford University, Los Angeles, and Stanford University, California

Communicated by C. TRUESDELL\*

1. Introduction. With the publication of Newton's *Principia* in 1687, classical particle mechanics received its first systematic formulation. In the nearly three centuries that have elapsed since that date, the foundations of this discipline have been re-examined and restated a great number of times<sup>2</sup>. From the standpoint of logical rigor and precision, however, none of the existing treatments of classical mechanics seems to be entirely free from serious defects: none of them comes even close to satisfying the standards set, let us say, by Hilbert in his axiomatization of Euclidean geometry<sup>3</sup>.

<sup>\*</sup> The communicator is in complete disagreement with the view of classical mechanics expressed in this article. He agrees, however, that strict axiomatization of general mechanics—not merely the degenerate and conceptually insignificant special case of particle mechanics—is urgently required. While he does not believe the present work achieves any progress whatever toward the precision of the concept of force, which always has been and remains still the central conceptual problem, and indeed the only one not essentially trivial, in the foundations of classical mechanics, he hopes that publication of this paper may arouse the interest of students of mechanics and logic alike, thus perhaps leading eventually to a proper solution of this outstanding but neglected problem.

<sup>&</sup>lt;sup>1</sup> We are grateful to Professor Herman Rubin, Professor Alfred Tarski, and Mr. Robert Vaught for a number of helpful suggestions in connection with this paper.

<sup>&</sup>lt;sup>2</sup> Among the works of the eighteenth and nineteenth centuries, those of Lagrange, Hertz, and Mach are outstanding. For more recent works, see Marcolongo [1], Hamel [1], [2], and [3], and Simon [1]; (the numbers in square brackets refer to items in the Bibliography). The references mentioned are, however, only a small part of the enormous literature in physics, mathematics, and philosophy concerning the foundations of mechanics; in fact, nearly every textbook in the subject attempts something in this direction.

<sup>&</sup>lt;sup>3</sup> Thus, although Hamel's formulation of mechanics is rightly regarded as one of the clearest existing treatments of the subject, we find in Hamel [2] (p. 3), the follow-