

*A Generalization of the Method of Separation of Variables**

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Introduction. In the application of the method of separation of variables to a partial differential equation for one unknown real function u of two independent real variables x, y one seeks for solutions of the form $u = X(x)Y(y)$ where $X(x), Y(y)$ are real functions¹. For example, if the method is applied to Laplace's equation $u_{xx} + u_{yy} = 0$ we are led to the identity

$$(1) \quad X'' Y + XY'' \equiv 0,$$

which implies $X''/X = -Y''/Y = k^2 = \text{const.}$, to reduce the determination of solutions of the stipulated form to the integration on the ordinary differential equations

$$(2) \quad X'' - k^2 X = 0, \quad Y'' + k^2 Y = 0.$$

There is, however, another line of argument, based on the identity (1), which opens the way to a generalization of the method. Actually (1) is a special case of the identity

$$(3) \quad x_1(x)y_1(y) + x_2(x)y_2(y) \equiv 0,$$

which we lend geometrical significance by introducing the curves

$$M: \quad x_1 = x_1(x), \quad x_2 = x_2(x); \quad N: \quad y_1 = y_1(y), \quad y_2 = y_2(y),$$

and framing the following question.

* This work was supported by the Office of Naval Research, and was presented in part at the Conference on Differential Equations held at the University of Maryland, September 8-9, 1950.

¹ See, however, §3 at the end of this paper.