

# *On the Propagation of Waves in Isotropic Incompressible Perfectly Elastic Materials\**

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*Communicated by C. TRUESDELL*

**1. Introduction.** The problem of the propagation of waves in incompressible, perfectly elastic solids has received little attention, though several authors, including Duhem [1], Hadamard [2], and Tolotti [3], have discussed the corresponding problem for compressible materials. Some important results recently obtained in the theory of large deformations of perfectly elastic materials, namely those of Rivlin [4], Green & Shield [5], apply only to incompressible materials. Furthermore, the theory has been applied most successfully to rubber, which can be considered incompressible. In view of these facts, it seems reasonable to consider the propagation of waves in such materials. When possible, we use Truesdell's notation [6].

At time  $t = 0$  let the points of a continuum be assigned coordinates  $X^\alpha$  in a stationary Euclidean coordinate system with metric tensor  $G_{\alpha\beta}$ . At any other time  $t$  assign them coordinates  $x^i$  in any independently selected coordinate system of the same type with metric tensor  $g_{ij}$ . Greek indices always refer to the coordinates  $X^\alpha$ , Latin to the coordinates  $x^i$ . It is assumed that the mappings

$$(1.1) \quad x^i = x^i(X^\alpha, t)$$

defining the motion of the continuum are single-valued, continuous, and satisfy certain differentiability conditions which will be stated precisely later. It is further assumed that, at each time  $t \geq 0$ , equations (1.1) possess a unique inverse  $X^\alpha = X^\alpha(x^i, t)$ . The symbol  $D/Dt$  denotes time differentiation with the variables  $X^\alpha$  held constant,  $\partial/\partial t$  time differentiation with the variables  $x^i$  held constant,

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\* Prepared under Army contract DA-33-008 ORD-454.

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