

*Integration Formulae and Boundary Conditions for the
Hyperbolic Equation with Three Independent
Variables and Regions Interior to the Cone**

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Communicated by T. Y. THOMAS

The use of an integrating factor in solving partial differential equations originated with Green in his study of the Laplace equation. In extending Green's ideas to the hyperbolic equation for $n = 2$, Riemann discovered that characteristics should partially bound the region of integration and that in general the Green potential involves a non-Euclidean distance. Hadamard¹ extended the Green potential to the general linear equation with variable coefficients.

Curiously enough waves in one and three space dimensions were treated by a Green method much earlier than waves in two space dimensions. No doubt one of Volterra's² great contributions was overcoming the difficulty of divergent integrals that are inherent in the latter problem. His method involved integrating the Green potential in a time-like direction, thus distributing the infinity over the axis of the cone. He was then able to enclose the axis of the cone by a cylinder in the Green manner without encountering divergent integrals. The integrating factor used in this paper is obtained by integrating the potential over a surface, thus eliminating the infinity entirely and replacing it by a finite jump discontinuity in one of the partial derivatives.

Integration of potentials naturally suggests the use of integration by parts in the differentiation of improper integrals. The application of differential opera-

* This paper is a portion of a dissertation submitted in partial fulfillment of requirements for the degree of Doctor of Philosophy at the University of Texas.

¹ See **II**. (Bold face Roman numerals refer to the bibliography.)

² See **X**.