

On Plane and Axially Symmetric Free Boundary Problems

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1. Recently a series of hydrodynamic comparison theorems showing qualitative dependence of flow speed on certain variations of the flow region have been applied with great success in the theoretical treatment of the classical free boundary problem.¹ In particular, these theorems have been instrumental in reducing the uniqueness problem to comparatively simple geometric considerations [1, 2, 5], and in materially extending the existence theory [4]. In this paper we shall make further application of the comparison theorems in presenting results of a qualitative nature for the major problems of free boundary theory, namely that of the cavity (finite or infinite) behind a given obstacle and of the free jet issuing from a channel. In general our results will be valid for both the plane symmetric and the axially symmetric situations, inasmuch as the comparison theorems upon which they are based have the same form in either case.

Specifically, §2 deals with the geometry of free streamlines. Theorem 1 removes any doubt about possible “pathological” behavior; with its aid we show, for example, that a starlike obstacle or channel gives rise to a starlike free boundary, a result of importance in uniqueness theory.² Roughly speaking, Theorem 2 compares the shape of cavities (jets) formed by different obstacles (channels). As a consequence, we find an alternate method of establishing

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¹ Hydrodynamic comparison theorems seem to have originated in the work of LAV-RENTIEFF [2]. More recently they have been extended and simplified by GILBARG [1] and SERRIN [5, 6]. Specific reference to these theorems, of which constant use will be made, occurs in the body of the paper. Numbers in brackets refer to the bibliography.

² See, for example, GILBARG [1].