

On Certain Special Solutions of the Φ -equation of Statistical Hydrodynamics

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In a previous paper‡ one of the authors considered the evolution in time of phase distributions of an unbounded incompressible fluid of mean velocity zero. Each such phase distribution is characterized by its characteristic functional $\Phi = \Phi(y(x), t)$ which is a functional of an arbitrary vector field

$$y(x) = \{y_1(x_1, x_2, x_3), y_2(x_1, x_2, x_3), y_3(x_1, x_2, x_3)\}$$

in x -space and a function of the time t . The important statistical quantities, the correlations of the velocity components at different space points, are obtained from Φ by functional differentiation. In place of the x -space occupied by the fluid its Fourier dual, the k -space, was introduced. The argument vector field is then an arbitrary complex-valued continuous vector field

$$z(k) = \{z_1(k_1, k_2, k_3), z_2(k_1, k_2, k_3), z_3(k_1, k_2, k_3)\}$$

with the property that

$$z^*(k) = z(-k).$$

It is supposed to vanish sufficiently rapidly at $k = \infty$ and at $k = 0$. A vector field $z(k)$ possesses a "solenoidal part" $\bar{z}(k)$ defined by

$$\bar{z}(k) = z(k) - \frac{k \cdot z(k)}{k^2} k$$

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‡ E. HOPF, "Statistical hydromechanics and functional calculus." This *Journal* Vol. 1, No. 1 (1952).