

*The Structure of Shock Waves in the Continuum Theory of Fluids**

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1. Introduction and summary. The purpose of this paper is to explore the consequences and potentialities of continuum methods in the theory of the steady one-dimensional shock profile. The problem of shock structure is to present a quantitative description of the narrow transition region in the shock wave. Special interest attaches to the problem because this region is so narrow as to call into question the basic concepts of continuum mechanics and the validity of the continuum equations. It will be our conclusion to reject as unfounded on the basis of present evidence the frequent assertion that the equations of fluid dynamics are either of little value or in some sense inapplicable to the shock problem.¹

From the time of Becker's fundamental paper [2] which seemed to show that the Navier-Stokes equations predict shock thicknesses so small as to invalidate continuum methods the belief has grown that only the kinetic theory is capable of a correct account of the shock structure. While other investigators, starting with Thomas [3], have emphasized the considerable effect on the shock thickness of temperature dependence of viscosity and thermal conductivity—these effects having been neglected in Becker's work—the feeling has persisted that

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¹ As examples of such assertions, GRAD ([1], p. 261) refers to the "breakdown of the Navier-Stokes equations", TSIEN (Mathematical Reviews **13**, p. 196) states that "early investigators . . . pointed out the inadequacy of the Navier-Stokes equations," LIGHTHILL (Mathematical Reviews **12**, p. 139) comments that "within stronger shocks the Navier-Stokes equations are too inaccurate . . .," and OSWATITSCH (Zentralblatt für Mathematik **44**, p. 216), commenting on shock calculations with the Navier-Stokes equations, states "In Anbetracht, daß es sich aber dabei bekanntlich nicht mehr um ein Problem der Kontinuitätsphysik handelt, sind solche Rechnungen kaum von praktischer Bedeutung."