

*Correction to the paper "On an Inequality for
Minimal Surfaces $z = z(x, y)$ ".**

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The lemma of Heinz quoted under b) in the author's note states: If $x = x(\alpha, \beta)$, $y = y(\alpha, \beta)$ is a continuous one-to-one mapping of $\alpha^2 + \beta^2 \leq 1$ upon the full set $x^2 + y^2 \leq 1$, if $x(0, 0) = y(0, 0) = 0$ and if x, y are harmonic in $\alpha^2 + \beta^2 < 1$ then

$$\Lambda(x, y) = x_\alpha^2 + x_\beta^2 + y_\alpha^2 + y_\beta^2 \Big|_{\alpha=\beta=0}$$

has a positive infimum for all those mappings.

On using the Poisson formula which expresses each of those mappings in terms of the boundary mapping $\omega = \omega(\varphi)$, $\varphi = \text{angle on } \alpha^2 + \beta^2 = 1$, $\omega = \text{angle on } x^2 + y^2 = 1$, Heinz transformed his lemma into the following equivalent statement.

$$\Lambda(\omega) = \left| \frac{1}{\pi} \int_0^{2\pi} e^{i\omega(\varphi)} \cos \varphi \, d\varphi \right|^2 + \left| \frac{1}{\pi} \int_0^{2\pi} e^{i\omega(\varphi)} \sin \varphi \, d\varphi \right|^2$$

(which is $\equiv \Lambda(x, y)$) has a positive infimum for all continuous and monotonic functions $\omega(\varphi)$ with the properties that $\omega(\varphi + 2\pi) \equiv \omega(\varphi) + 2\pi$ or $\equiv \omega(\varphi) - 2\pi$ and that

$$\int_0^{2\pi} e^{i\omega(\varphi)} \, d\varphi = 0.$$

At the end of his note the author stated the conjecture that $\inf \Lambda = 16/\pi^2$. This value is attained by the degenerate harmonic mapping which maps $\alpha^2 + \beta^2 \leq 1$ upon a diameter of $x^2 + y^2 \leq 1$ or, equivalently, by the degenerate

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