

Abel and Riesz Transforms of Series Having Bounded Partial Sums

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1. Introduction. It is our purpose to obtain best estimates relating the Abel and Riesz transforms

$$(1.1) \quad A(t) = \sum_{k=0}^{\infty} t^k u_k, \quad R^{(r)}(n) = \sum_{k=0}^n \left(1 - \frac{k}{n+1}\right)^r u_k$$

of series $\sum u_n$ of complex terms satisfying the condition

$$(1.2) \quad \limsup_{n \rightarrow \infty} |s_n| < \infty$$

where

$$(1.21) \quad s_n = \sum_{k=0}^n u_k \quad n = 0, 1, 2, \dots$$

is the sequence of partial sums of $\sum u_n$. Some of our work applies to Riesz transformations of arbitrary complex order, but we assume everywhere that r is real and $r \geq 0$; it is not assumed that r is an integer. Our problems are, roughly speaking, to determine relative values of t and n such that the quantity

$$(1.22) \quad |A(t) - R^{(r)}(n)|$$

will be small and to determine how small the quantity can be made by best choices of t and n .

Before formulating our problems, we need some terminology. If (1.2) holds, then the sequence s_n has a nonempty bounded set Z of limit points. There is a