

# *Nonexistence of Conformal Singularities in Solid Spaces\**

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The most interesting regular spots of an analytic conformal mapping of a complex variable are its winding points or conformal singularities. The same is true for a conformal (also called isothermic) harmonic representation of a minimal surface. However, in mapping a region of classical riemannian space of dimension  $n > 2$  conformally and analytically into another classical riemannian space it is customary to assume that the mapping, unless constant, be devoid of conformal singularities. This assumption is, as I shall show, superfluous: for it follows from the conformality and analyticity of the mapping together with the positive definite structure of classical riemannian space.

**Conformal mapping.** Let  $X$  be a region of real riemannian space of dimension  $n > 2$  with real symmetric positive definite metric tensor  $g_{\alpha\beta} = g_{\alpha\beta}(x)$  differentiable to all orders; and let  $Y$  be a real riemannian space with real symmetric positive definite metric which need not be specified. An analytic mapping  $y = y(x)$  from  $X$  into  $Y$  is called conformal if there exists on  $X$  a scalar function  $p = p(x)$  such that

$$y_{\alpha} y_{\beta} = p g_{\alpha\beta}$$

throughout  $X$ : covariant derivatives of  $y$  with respect to coordinates of  $x$  being denoted by subscripts and inner product in the space  $Y$  of a pair of derivatives of  $y$  by juxtaposition.

**Conformal singularity.** Since the metrics of  $X$  and  $Y$  are positive definite and

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