

Extensions in the Applicability of Riemann's Formula

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§1. PRELIMINARIES

Introduction. The object of this paper is to investigate the Cauchy initial values problem for the linear hyperbolic differential equation

$$L(u) \equiv u_{xy} + au_x + bu_y + cu = f$$

when the initial data has a finite number of discontinuities, when the tangent to the initial curve has a finite number of discontinuous changes in direction, and when some characteristics meet the initial curve in more than one point.* It will be assumed that a, b, c together with a_x, b_y are continuous functions of x, y in any particular characteristic rectangle R considered; further, for convenience, it will be supposed that $f \equiv 0$.

With these conditions on a, b, c , it is well-known that there exists a function $v(x, y; \xi, \eta)$, the Riemann function, which satisfies the adjoint equation

$$(1) \quad M(v) \equiv v_{xy} - (av)_x - (bv)_y + cv = 0,$$

as well as the conditions

$$(2) \quad \begin{cases} v_y = av & \text{on } x = \xi, \\ v_x = bv & \text{on } y = \eta, \\ v = 1 & \text{at } x = \xi, y = \eta. \end{cases}$$

* The classical literature states that there is no solution, in general, of this problem, see [2].