

Novel Uses of Functional Equations

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We derive functional equations related to certain topics in electromagnetic theory. Although many of the results are believed to be new, the interest centers chiefly in the method, which does not take Maxwell's equations as the point of departure.

PART I. STRATIFIED DIELECTRIC MEDIA

The equation for uniform dielectric media. If a plane sheet with complex reflection coefficient r and transmission coefficient t is immediately followed by a second sheet with coefficients R, T , then the coefficients for the combination are

$$(1) \quad \frac{r + (t^2 - r^2)R}{1 - rR}, \quad \frac{tT}{1 - rR}.$$

(Equation (1) may be established by superposition or by considering multiple reflection; see [2].)

Suppose, now, that a homogeneous sheet of thickness x is immediately followed by a similar sheet of thickness y . If they be made of the same material, the composite sheet is a homogenous sheet of thickness $x + y$. Applying (1) to this composite sheet, we obtain the functional equations

$$(2) \quad R(x + y) = \frac{R(x) + [T^2(x) - R^2(x)]R(y)}{1 - R(x)R(y)}$$

$$(3) \quad T(x + y) = \frac{T(x)T(y)}{1 - R(x)R(y)},$$