

*On the Asymptotic Stability of Solutions of Parabolic Differential Equations**

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1. Introduction. We consider in this paper the stability of solutions of a wide class of nonlinear parabolic differential equations having the following general form:

$$(1.1) \quad \frac{\partial u_\nu}{\partial t} = \sum_{i,j} a_{ij}(x) \frac{\partial^2 u_\nu}{\partial x_i \partial x_j} + \sum_i b_i(x) \frac{\partial u_\nu}{\partial x_i} + F(x, t, u_\mu),$$
$$i, j = 1, \dots, n; \quad \nu, \mu = 1, \dots, m; \quad x \in G; \quad t > 0.$$

The domain of definition G is a bounded open region in n -dimensional x -space. Suitable conditions are to be imposed on the coefficients $a_{ij}(x)$, $b_i(x)$ and on the nonlinear part F . The solution vector $U(x, t)$ besides satisfying certain obvious continuity and differentiability conditions is assumed to be zero on the boundary Γ of G for $t \geq 0$. It is then shown that for suitably "small" initial values the solution vector tends to zero as t tends to infinity, *i.e.*,

$$\lim_{t \rightarrow \infty} |u_\nu(x, t)| = 0 \quad \text{uniformly in } x, \quad \nu = 1, \dots, m.$$

These details will be made more precise subsequently in the actual statement of the theorems.

The notion of stability of solutions of partial differential equations as used here is a generalisation of the classical definition of stability as applied to solu-

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