

*Asymptotic Solution of Linear Hyperbolic Partial Differential Equations**

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1. Introduction. In view of the difficulties in obtaining exact solutions of partial differential equations with prescribed initial and boundary conditions, attention has been given to the possibilities of obtaining solutions in the form of asymptotic series. The present paper presents a method of obtaining an asymptotic solution for some types of problems involving linear second order hyperbolic partial differential equations in n independent variables. Specifically, we shall consider the equation

$$(1.1) \quad L(u) \equiv \sum_{i,j=1}^n a^{ij} u_{ij} + \sum_{k=1}^n b^k u_k + cu = f_{it},$$

where u is a function of x_1, x_2, \dots, x_n , the coefficients in $L(u)$ are functions of x_1, x_2, \dots, x_{n-1} , the superscripts serve merely for identification, and $u_{ij} = \partial^2 u / \partial x_i \partial x_j$. For brevity we shall write x for x_1, x_2, \dots, x_{n-1} and t for x_n so that $u(x_1, x_2, \dots, x_{n-1}, x_n) = u(x, t)$. The source function $f(x, t)$ is assumed to be of the form $g(x)h(t)$, and for the most important results $h(t)$ will be limited to $e^{-i\omega t}$ where ω is the circular frequency of the source.

It will be shown that, apart from a transient which can be neglected for large t , the solution of equation (1.1) takes the form $u(x)e^{-i\omega t}$. We shall then obtain for $u(x)$ an asymptotic series whose terms contain the zero and positive integral powers of $1/\omega$. A method of obtaining the coefficients of this asymptotic series

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