

# *Generalizations of the Inversion Formula of Thin Airfoil Theory\**

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1. In the theory of two-dimensional flow of an ideal fluid past a thin airfoil there arise two types of problem, which may be called the "thickness" problem and the "lifting" problem; they lead to two different types of boundary value problem for the complex velocity  $w = u - iv$  in the complex plane of flow,  $z = x + iy$ . The problem in the "thickness" case is the determination of the flow past a symmetrical airfoil at zero angle of attack, leading to the following mathematical formulation: along a portion of the real axis (the chord:  $a_1 \leq x \leq a_2$ , say) values of the velocity component  $v$ , equal in magnitude and opposite in directions for the upper and lower sides of the chord, are prescribed:  $v(x, +0) = -v(x, -0)$  ( $=v_0$ , say). An analytic function  $w$  whose imaginary part satisfies this boundary condition and for which  $w = 0$  at infinity is to be determined. Throughout this paper, the letter  $w$  represents only the "disturbance" velocity, which must vanish at infinity. Of special interest is the value of the  $u$ -component on the chord,  $u(x, 0)$ .

Since the boundary condition for  $v$  is antisymmetric with respect to the real axis, it is easily recognized that the harmonic function  $v$  is itself antisymmetric:  $v(x, y) = -v(x, -y)$ . It follows that for the portions of the real axis outside the chord we have  $v(x, 0) = 0$ . Thus, the problem may be formulated as a boundary value problem for the upper half plane (say), with  $v$  given along the whole real axis.

The solution is immediately obtained by distributing along the real axis sources whose strength per unit length equals twice the velocity  $v_0$  on the upper

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