

Some Implications of Liapunov's Conditions for Stability

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1. Introduction. A. M. Liapunov [9] established several criteria for the stability of systems of ordinary differential equations which are independent of considerations of variational equations, and which form the basis of what Liapunov himself called the "second method." This method gave rise to a development of the theory of stability, to a large extent due to I. G. Malkin [10–16] and K. Persidskii [21–23] and numerous other Russian mathematicians, which has come into prominence anew in recent years.

In this paper we point out some implications of this theory of stability concerning linear as well as certain non-linear systems of differential equations and bring together more closely a number of related results which seem to have escaped attention thus far. In particular, we establish for the general linear system the equivalence of the existence of what is commonly called a Liapunov function with certain conditions upon any set of n linearly independent solutions. This not only demonstrates the strong type of stability that is implied by the existence of such a Liapunov function, but also represents the connecting link between

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