

*Parabolic Cylinder Functions of Large Order*¹

A. ERDÉLYI, M. KENNEDY, & J. L. MCGREGOR

California Institute of Technology, Pasadena, California

Communicated by C. TRUESDELL

1. Introduction. In this paper we shall investigate the solutions of the differential equation

$$(1.1) \quad \frac{d^2 y}{dx^2} - 4\nu^2(x^2 - 1)y = 0$$

when ν is a large (real or complex) parameter, and x is an unrestricted complex variable. This equation and closely related differential equations occur in problems of wave motion inside or outside a parabolic cylinder, electromagnetic waves in an inhomogeneous atmosphere, diffraction problems, vibration problems, and as the Schrödinger equation of the harmonic oscillator in quantum theory. In several of these problems it is of importance to know the behavior of the solutions for large ν . Moreover, (1.1) appears to be the simplest differential equation of the second order with two transition points and would seem the natural comparison equation for studying the behavior for large ν of solutions of an equation of the form

$$(1.2) \quad \frac{d^2 y}{dx^2} - 4\nu^2 \phi(x)y = 0$$

in a region which contains two simple zeros, or one double zero, of $\phi(x)$ very much in the same way as the Airy equation (4.1) is the comparison equation when the region includes only one transition point.

The differential equation (1.1) with a large parameter ν has been studied by a number of writers among whom it will be sufficient to mention Watson (1918), Kienast (1936), and Schwid (1935). When $\nu = n + \frac{1}{2}$, where n is a non-negative

¹ Prepared under contract Nonr-220(11) between the Office of Naval Research and the California Institute of Technology. Reference No. NR 043-121.