

# *Essential Singularities of Solutions of a Class of Linear Partial Differential Equations in Three Variables\**

STEFAN BERGMAN

Stanford University, Stanford, California

**I. The method of integral operators and its application for the study of pole-like singularities of the first kind and essential singularities.** Various problems of importance in physical applications require development of a theory of multi-valued solutions and of singularities of functions satisfying linear partial differential equations. When developing this theory in the case of differential equations in two variables†

$$(1.1) \quad \begin{aligned} \frac{\partial^2 \tilde{\psi}}{\partial y^2} + \frac{\partial^2 \tilde{\psi}}{\partial z^2} + \tilde{F}\tilde{\psi} &= -\frac{\partial^2 \psi}{\partial Z \partial Z^*} + F\psi = 0, \\ Z &= \tfrac{1}{2}(z + iy), \quad Z^* = -\tfrac{1}{2}(z - iy), \\ \tilde{\psi} \equiv \tilde{\psi}(z, y) &= \psi(Z, Z^*) \equiv \psi, \quad \tilde{F} = 4F \end{aligned}$$

where  $F = F(Z, Z^*)$  is an entire function, we associate with  $\psi$  the analytic function  $\psi(Z, 0)$  of one complex variable  $Z$ . The integral operator which transforms the function  $\psi(Z, 0)$  into that complex solution of (1.1) whose real part is

---

\* This work was performed under a contract with the Office of Naval Research.

† We use here a somewhat unusual complex notation  $Z$  and  $Z^*$  (instead of, say,  $\zeta = y + iz$ ,  $\bar{\zeta} = y - iz$ ) since the notation  $(x, Z, Z^*)$  has been used in the case of three variables in a number of previous papers and we wish to avoid modifications when comparing our results with various formulas used in the previous papers. In the case of two variables we employ usually the variables  $Z = \tfrac{1}{2}(z + iy)$  and  $\bar{Z} = -Z^* = \tfrac{1}{2}(z - iy)$ , so that for  $z$  and  $y$  real,  $Z$  and  $\bar{Z}$  are conjugate to each other and

$$\frac{\partial}{\partial Z} = \frac{1}{2} \left( \frac{\partial}{\partial z} - i \frac{\partial}{\partial y} \right), \quad \frac{\partial}{\partial Z^*} = -\frac{1}{2} \left( \frac{\partial}{\partial z} + i \frac{\partial}{\partial y} \right).$$