

A Theorem on Rearrangements and its Application to Certain Delay Differential Equations

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1. Introduction. Let $f(x)$ be real valued and possess a continuous derivative $f'(x)$ for all real x such that everywhere

$$(1.1) \quad f(x) > 0, \quad f'(x) > 0, \quad \text{and} \quad f(0) = 1.$$

Under the conditions (1.1) we consider over $t > 0$ the necessarily nonlinear delay differential equation

$$(1.2) \quad x'(t) = -\frac{c}{\theta} \int_0^\theta (\theta - h) \{f(x(t-h)) - 1\} dh$$

for given constants $c > 0$ and $\theta > 0$. We remark (theorem 7) that a unique solution of (1.2) over $t > 0$ is determined by every real valued, continuous function over $-\theta \leq t \leq 0$. Define over $t > 0$

$$(1.3) \quad [R(y)](t) = F(y(t)) - \left[\frac{1}{\theta} \int_0^\theta f(y(t-h)) dh \right] y(t) + \frac{1}{2c} [y'(t)]^2,$$

where $F(y) = 1 + \int_0^y f(z) dz$. Then we shall find (lemma 3) that every solution $x(t)$ of (1.2) must also satisfy for $t_2 > t_1 > \theta$

$$(1.4) \quad \int_{t_1}^{t_2} x(t) \{f(x(t)) - f(x(t-\theta))\} dt = -\theta \{[R(x)](t_2) - [R(x)](t_1)\}.$$