

Finite Plane Strain for Orthotropic Bodies

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1. Introduction. Some aspects of large elastic deformations of homogeneous anisotropic materials have been considered by Ericksen & Rivlin [1]. In particular they solve completely a number of problems for an incompressible material which is transversely isotropic with respect to a given direction. In this paper we consider the problem of finite plane strain for orthotropic and transversely isotropic materials. The flexure of a cuboid is then partly solved in terms of a general strain energy function for compressible orthotropic materials; the problem is solved completely for incompressible orthotropic materials. The finite extension and inflation of a tube, which is transversely isotropic with respect to its axis, is partly solved in terms of a general strain energy function for compressible materials; this problem was solved completely for incompressible materials by Ericksen & Rivlin [1].

2. Notation. Various equivalent forms of the natural state theory of elasticity have been discussed in a paper by Truesdell [2]. Here we summarize the notation we shall use in this paper. The points of an unstrained and unstressed body at time $t = 0$ are defined by a system of rectangular Cartesian coordinates x_i or by a general curvilinear system of coordinates θ_i . The curvilinear coordinates θ_i move with the body as it is deformed and form a curvilinear system in the strained body at time t . The covariant and contravariant metric tensors for the coordinate system θ_i in the unstrained body are denoted by g_{ij} and g^{ij} respectively, and for the coordinate system θ_i in the strained body, at time t , the corresponding metric tensors are G_{ij} and G^{ij} respectively. We write

$$g = |g_{ij}|, \quad G = |G_{ij}|, \quad (2.1)$$

Latin indices taking the values 1, 2, 3. The covariant strain tensor is γ_{ij} where

$$\gamma_{ij} = \frac{1}{2}(G_{ij} - g_{ij}). \quad (2.2)$$