

Non-linear Elliptic Equations Without Non-linear Entire Solutions¹

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§1. Introduction. A) The equation of minimal surfaces

$$(1 + \varphi_y^2)\varphi_{xx} - 2\varphi_x\varphi_y\varphi_{xy} + (1 + \varphi_x^2)\varphi_{yy} = 0$$

occupies a special position in the theory of quasi-linear elliptic equations since, just like its linear analogue, the Laplace equation, it can be investigated by methods of complex function theory. In particular, the following properties of solutions of this equation are known.

a. *Every entire solution is a linear function.* (By an entire solution we mean here and hereafter one which is defined for all values of x and y .)

b. *A single-valued solution defined in a neighborhood of the point at infinity is always such that its gradient attains a finite limit at infinity. The same is true for a multiple-valued solution, provided that its gradient is finitely many-valued.*

c. *At a finite point a single-valued solution can have at most a removable isolated singularity.*

B) Theorem **a** is due to S. Bernstein [2],² who obtained it as a corollary of his celebrated geometric theorem. The latter theorem implies that every elliptic equation of the form

$$A\varphi_{xx} + 2B_{xy} + C\varphi_{yy} = 0$$

possesses only constant entire bounded solutions. Other proofs of Bernstein's theorem on minimal surfaces were given by T. Rado [19], myself [3], and E. Heinz [13]. I proved Theorems **b** and **c** by the same method as Bernstein's

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² Numbers in brackets refer to the bibliography. Cf. also HOPF [14], MICKLE [18].