

# *On a Problem of Type, with Application to Elliptic Partial Differential Equations\**

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1. Let  $\varphi(x, y)$  be a function of class  $C^{(\prime\prime)}$ , defined for all finite real values of  $x$  and  $y$ . The function  $\varphi(x, y)$  then represents a surface  $S$  over the  $(x, y)$  plane which has a simply covered projection on this plane. It is known that for any such surface  $S$ , there will exist a complex valued function  $f(x, y)$  which maps the whole of  $S$  in a 1-1 conformal manner onto a simply-connected open set  $T$  of the plane. Loewner has raised the question whether there exist surfaces  $S$  such that  $T$  will have a finite boundary point. No such surfaces have been discovered. Under certain circumstances, *e.g.* if  $S$  is a surface of revolution or if  $(\varphi_x^2 + \varphi_y^2)^{\frac{1}{2}} < K \log (2 + x^2 + y^2)$  for some constant  $K$ , it is easy to show that  $T$  covers the plane. It has been shown by Blanc & Fiala [4]<sup>1</sup> that if the surface integral of the Gaussian curvature of  $S$  is bounded below, then  $T$  must cover the plane. It is easy to construct surfaces  $S$  for which the surface integral of the Gaussian curvature is not bounded below, but such that  $T$  covers the plane.

In the present paper we consider surfaces  $S$  which are defined by solutions of variational problems

$$(1) \quad \delta \iint F(p^2 + q^2) dx dy = 0, \quad p = \varphi_x, \quad q = \varphi_y.$$

In addition to the usual requirement of ellipticity,

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<sup>1</sup> Numbers in brackets refer to references at end of paper.