

*On the Concept of Concentrated Loads and an Extension of the Uniqueness Theorem in the Linear Theory of Elasticity\**

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**1. Introduction.** The traditional formulation of the second boundary-value problem of the linear theory of elasticity in the presence of concentrated surface loads rests on the following properties required of the solution to such a problem: (a) *it must satisfy the field equations of the theory throughout the region occupied by the medium;*<sup>1</sup> (b) *it must conform to the boundary conditions for distributed surface tractions;*<sup>2</sup> (c) *it must be regular*<sup>3</sup> *with the exception of singularities at the points of application of concentrated loads such that the resultant of the tractions on any surface surrounding a given load-point, and lying wholly in the body, tends to the corresponding prescribed concentrated load in the limit as the surface is contracted toward the load-point.*

Since the classical uniqueness theorem does not hold in the presence of singularities of the type under consideration, there is no assurance that the foregoing formulation uniquely characterizes the solution to a concentrated-load problem. That this is not merely an idle concern has been shown previously. In [1]<sup>4</sup> were exhibited an infinite aggregate of distinct "solutions" corresponding to the half-space and the sphere under normal concentrated loads, each of which possesses

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<sup>1</sup> Internal concentrated loads are excluded for the time being.

<sup>2</sup> If, in particular, the loading consists of concentrated forces only, the solution must clear the boundary from tractions.

<sup>3</sup> The precise nature of these regularity requirements, which is ordinarily not specified, will be considered later in detail.

<sup>4</sup> Numbers in brackets refer to the bibliography at the end of this paper.